Geometric Scaling at RHIC and LHC

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Outline

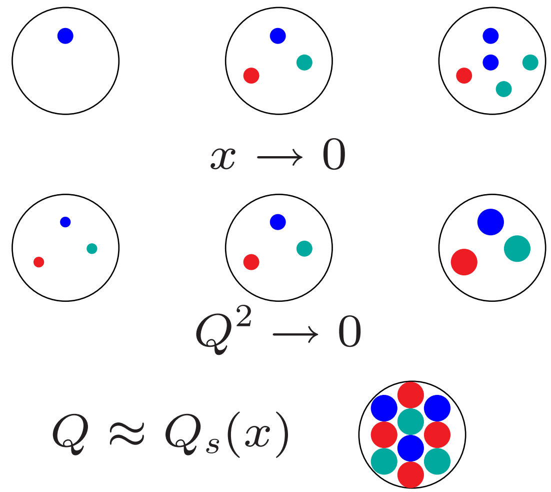
Situation:
Successful description of DIS data using (geometrical scaling) dipole models
Question:
Also possible for RHIC data?

1. **Introduction**
   - DIS and the dipole picture
   - Geometric scaling in DIS
2. **The dipole picture for hadron production at hadron colliders**
   - Modeling the dipole cross section and geometric scaling
   - What to expect from BFKL (BK) evolution
3. **Results**
   - Scaling at RHIC
   - Possible conclusions for small-x evolution
4. **LHC predictions**
   - Probing smaller x
5. **Conclusions**

1. Introduction

- eP-scattering at HERA: Strong **rise** of the gluon distribution $f(x, Q^2)$ at small x
  - Rise of distrib. $f(x, Q^2)$ due to softer gluon emission
  - **Problem**: Undamped rise may violate unitarity (Froissart bound)
  - **Reason: Linear** DGLAP or BFKL eqs.: non-interact. partons in the proton

- Partons start to overlap $\Rightarrow$ becomes important
- Number of partons rises with $x \longrightarrow 0$
  "size" $\sim \frac{1}{Q}$ of partons rises with $Q^2 \longrightarrow 0$
  Interaction becomes important for $Q \lesssim Q_s(x)$
- $\Rightarrow$**New relevant scale at small x: $Q_s(x)$**

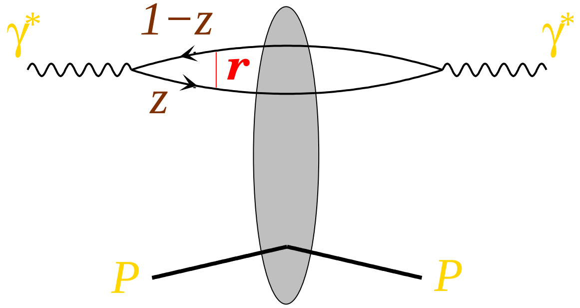


- **Interaction** between partons $\Rightarrow$ non-linear corrections to the evolution equations
  [Gribov, Levin & Ryskin '81-'83]

- **Idea**: Interaction $\Rightarrow$ rise of the gluon distribution at small x is tamed
  $\Rightarrow$ gluon distribution "saturates"

Color-Dipole Picture

- Investigation of small-x saturation most transparent in the **color-dipole picture**:



z: longitudinal photon momentum fraction carried by the quark

$\boldsymbol{r}$: transverse $(q\bar{q})$-size

[Nikolaev & Zakharov '90; Mueller '94]

- **Intuitive in the P-rest frame:** for small x, γ^* fluctuates mainly into $q\bar{q}$-dipole where $\tau_{q\bar{q}-\mathrm{formation}} \gg \tau_{(q\bar{q})\,P-\mathrm{interaction}} \Rightarrow$ factorization:

$$\sigma_{L,T}(x, Q^2) = \int_0^1 dz \int d^2\boldsymbol{r}\, |\Psi_{L,T}^{\gamma^*\to q\bar{q}}(z, r; Q^2)|^2\, \sigma_{\mathrm{DP}}(r = |\boldsymbol{r}|, x)$$

- **Photon wave function,** $\Psi_{L,T}^{\gamma^*\to q\bar{q}}$: perturbatively calculable

- **Dipole-proton cross section** σ_{DP} contains non-perturbative elements (proton):
  - Simplest approach in the framework of pQCD: two-gluon exchange

$$\sigma_{\mathrm{DP}}(r, x) = \frac{\pi^2}{3}\alpha_s\, xG(x, \mu^2)\, r^2 + \mathcal{O}(r^4)\,, \qquad \sigma_{\mathrm{DP}} \Leftrightarrow \text{gluon distrib.}$$

  - $r \gtrsim 1/Q_s(x)$: σ_{DP} saturates towards a black disc limit $\sigma_0 \approx \pi R_h^2$

Parameterizing the dipole cross section

- HERA data on structure function F_2 at low x ($x \lesssim 0.01$) quite well described by [Golec-Biernat, Wüsthoff]

$$\sigma_{\mathrm{GBW}}(r, x) = \sigma_0 \left\{ 1 - \exp\left[-\frac{1}{4} r^2 Q_s^2(x) \right] \right\}$$

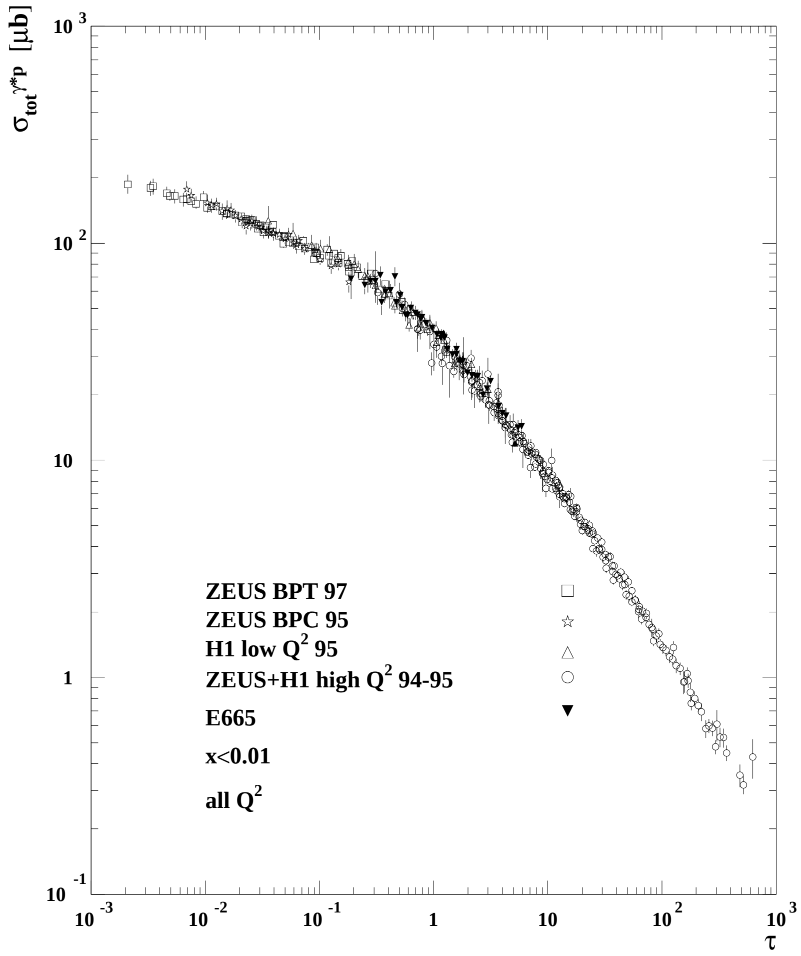
  - r denotes the transverse size of the dipole
  - x dependence of the saturation scale:

$$Q_s(x) = 1\,\mathrm{GeV}\left(\frac{x_0}{x}\right)^{\lambda/2}, \text{ where } x_0 \simeq 3 \times 10^{-4} \text{ and } \lambda \simeq 0.3$$

  Consistent with NLO BFKL evolution, which gives $Q_s^2(x) \sim 1/x^\lambda$ with $\lambda \simeq 0.3$ [Triantafyllopoulos, 2002].

Geometric scaling

- Basic feature of GBW model: geometric scaling $\sigma_{\text{DP}}(rQ_s) \;\Rightarrow \sigma_{\gamma^* p}(Q^2/Q_s^2(x))$

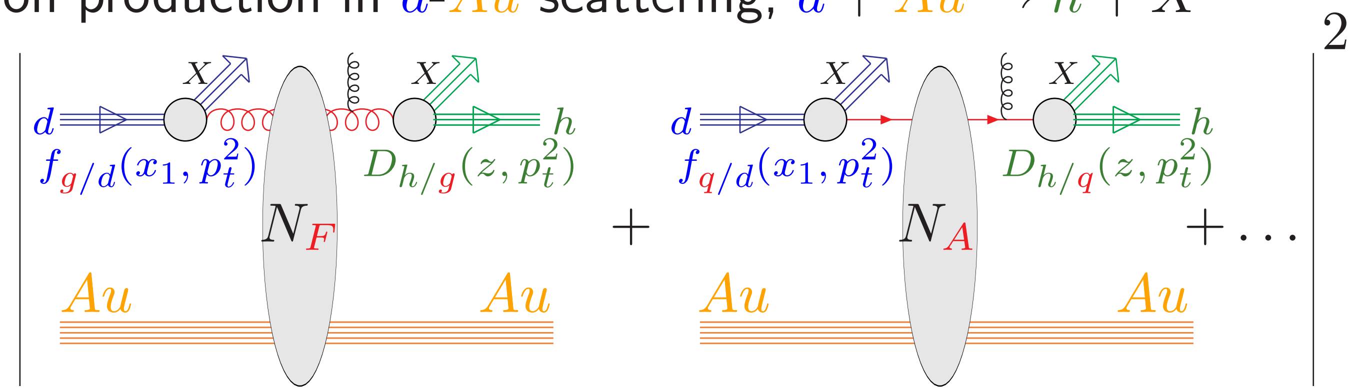


  - Indeed the DIS **data** depend only on $\tau = Q^2/Q_s^2(x)$ [Stasto, Golec-Biernat and Kwiecinski, '00]
  - Only true for **small** x data ($x < 0.01$)
  - The **whole** Q^2 **region** can be described (even the photo-production limit $Q^2 \longrightarrow 0$)
  - Scaling behavior is quite model independent
  - Feature holds also outside the saturation region
  - Seen as the strongest phenom. support for saturation

- **But** more precise data require at large Q^2 scaling violating modifications e.g. by taking DGLAP evolution into account [Bartels et al 2002], [Gotsman et al 2002]

2. Hadron production at hadron colliders in the dipole picture

- Hadron production in d-Au scattering, $d + Au \rightarrow h + X$



  - Amplitude: Wilson lines sum soft interact. of parton with nucleus (CGC)
    Squaring amplitude $\Rightarrow$ dipoles $N_{A,F}$ entering the cross sections

$$\Rightarrow \frac{\mathrm{d}N(dAu \rightarrow h(p_t, y_h)X)}{\mathrm{d}y_h\, \mathrm{d}^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^{1} dx_1 \frac{x_1}{x_F} [f_{q/d}(x_1, p_t^2)\, N_F(q_t, x_2)\, D_{h/q}(x_F/x_1, p_t^2)$$

[Dumitru & Jalilian-Marian 2006]

$$+ f_{g/d}(x, p_t^2)\, N_A(q_t, x_2)\, D_{h/g}(x_F/x_1, p_t^2)]$$

  - p_t, y_h: transv. momentum and rapidity of produced hadron ($x_F \equiv \frac{p_t}{\sqrt{s}} \exp[y_h]$)
  - $q_t = \frac{x_1}{x_F} p_t$: transverse momentum of dipole probing the target nucleus (CGC)
  - $x_2 = x_1 \exp[-2y_h]$: momentum fraction of the target partons
  - x_1: momentum fraction of the hard parton in the probe
  - Loop effects absorbed in DGLAP evolution of $f_{(q,g)/d}$ and $D_{h/(q,g)}$

Modeling the dipole scattering amplitudes $N_{A,F}$

- Dipole scattering amplitude following DHJ (adjoint repres. for gluon)

$$N_A(q_t, x_2) \equiv \int d^2r \, e^{i\,\vec{r}\cdot\vec{q}_t} \, N_A(r = |\vec{r}|, q_t = |\vec{q}_t|, x_2)$$

  - N_F (fundam. repres. for quarks) from N_A : $(r^2 Q_s^2)^\gamma \to (\frac{C_F}{C_A} r^2 Q_s^2)^\gamma$, $\frac{C_F}{C_A} = \frac{4}{9}$
  - Saturation scale, $Q_s^2(x) = A_{\rm eff}^{1/3} \left(\frac{x_0}{x}\right)^\lambda$, $\lambda = 0.3$, $x_0 = 3 \cdot 10^{-4}$, $A_{\rm eff} \approx 18.5$

- Ansatz for N_A introduced by modifying the GBW model ($\gamma = 1$):

$$N_A(r_r, q_t, x) = 1 - \exp\left[-\tfrac{1}{4}(r^2 Q_s^2(x))^{\gamma(r,x)}\right]$$

  - **Small r**: BFKL limit is recovered and γ is related to the anom. dimension:

$$N(r, x) \sim x\, g(x, \mu(r)^2) \quad \Rightarrow \quad \frac{d\, x\, g(x, \mu(r)^2)}{d\, \log x_0/x} \sim \gamma(r, x)\, x\, g(x, \mu(r)^2)$$

  - γ chosen to be a function of q_t rather than r $\Rightarrow$ simplifies Fourier transform.

Expectations on anomalous dimension γ

- **Expectations on $\boldsymbol{\gamma(r,x)}$ from small x evolution**
  - Linear BFKL evol. with satur. bound. cond. inspires $\gamma(q_t = Q_s) \approx 0.628 \equiv \gamma_s$
    e.g. [Iancu et al 2002, Mueller et al 2002, Triantafyllopoulos 2002]
  - However, not really a feature of the non-linear BK equation
    [Boer, Wessels, A.U. 2007]
  - Fixed x and $r \to 0$: $\gamma \to 1$ to reproduce the limit $N \sim r^2$
  - γ rises only logarithmically as $\frac{1}{y} \log q_t/Q_s$
- Good description of **forward** hadron production in $d+Au$ collisions at **RHIC** with [Dumitru et al 2006] similar to [Kharzeev et al 2004]

$$\gamma(q_t, x) = \gamma_s + (1-\gamma_s)\,\frac{\log(q_t^2/Q_s^2(x))}{\lambda y + d\sqrt{y} + \log(q_t^2/Q_s^2(x))}\,, \quad y = \log 1/x$$

  - γ depends explicitly (not only via Q_s) on $x \Rightarrow$ scaling violation
- **Questions we want to address**:
  - Are the central rapidity data also describable?
  - Are geometric scaling violations really required?
  - What to expect at LHC?

Our new model

- Our **parameterization of the anomalous dimension** γ

$$\gamma(w = q_t/Q_s(x)) = \gamma_1 + (1-\gamma_1)\,\frac{(w^a - 1)}{(w^a-1)+b}$$

  - γ_1: value at the saturation scale
  - a: defines how fast the limit 1 is reached for large w, $1-\gamma(w) \sim \frac{1}{w^a}$
  - Main differences to DHJ model: no scal. violation and steeper rise towards 1

- Leads to **faster fall off of the dipole scattering amplitude** with rising q_t

$$N_A(q_t) \approx \frac{2\pi}{q_t^2}\frac{1}{w^{2\gamma(w)}}\frac{1}{4}\int_0^\infty dz\, z\, \mathrm{J}_0(z)\,(-z^{2\gamma(w)}) = \frac{2\pi\, 2^{2\gamma(w)-1}\, Q_s^{2\gamma(w)}}{{q_t}^{2\gamma(w)+2}}\,\frac{\Gamma(1+\gamma(w))}{-\Gamma(-\gamma(w))}$$

$$\overset{\gamma(w)\to 1}{\approx} \frac{4\pi\, Q_s^2}{q_t^4}\,(1-\gamma(w)) \propto \begin{cases} \dfrac{Q_s^2}{q_t^4 \log(q_t^2/Q_s^2)} & \text{for DHJ } \gamma \\[2ex] \dfrac{Q_s^{2+a}}{{q_t}^{4+a}} & \text{for our scaling } \gamma \end{cases}.$$

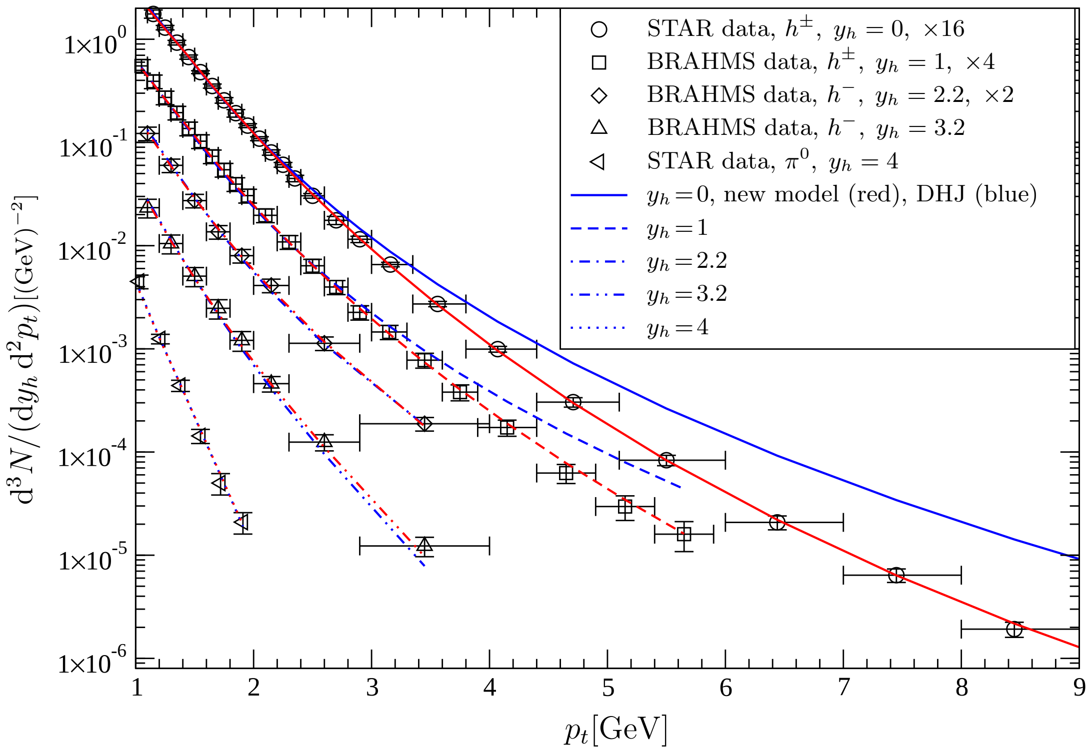
  - Folding with parton and fragment. func. $\Rightarrow$ steeper fall-off of p_t distribution

3. Results

- Note, due to folding with non-scaling pdf's and fragment. functions: **scaling dipole ampl.** $N(q_t/Q_s(x))$ **doesn't lead to scaling** p_t **distr.** $\mathrm{d}N(p_t/Q_s)$

- Taking $\gamma(q_t = Q_s) = \gamma_1 = 0.628$ and fitting parameter $a = 2.82$ and $b = 168$ $\Rightarrow$ very good description of RHIC data using a **scaling model**

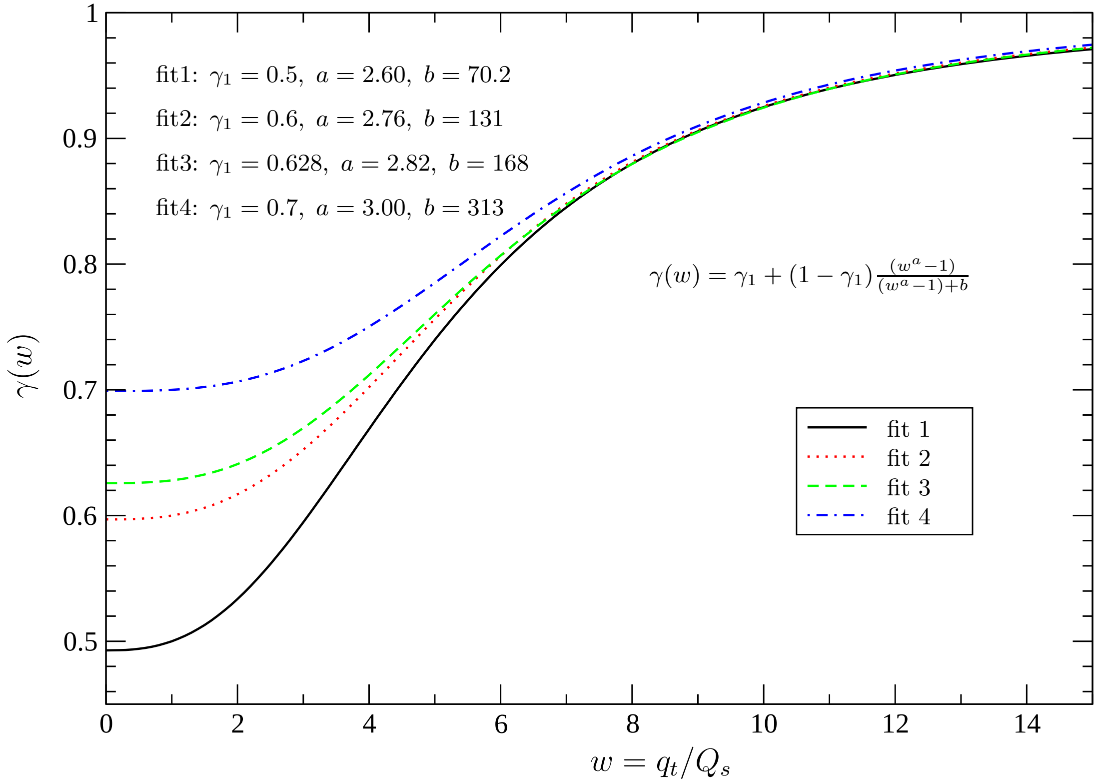
- For $y_h \approx 0-1$: DHJ model starts to fail for $p_t \gtrsim 2.5$ GeV
  - There: $x \gtrsim 0.01$
  - But: Q_s still larger than in DIS
- LO analysis requires K **factors**: drops from $K \approx 4$ to $K \approx 0.7$ between $y_h = 0$ and $y_h = 4$
  - NLO pQCD analysis suggests p_t independent K factors



- **RHIC data completely compatible with geometric scaling!**

Constraining γ

- Different sets of parameters are able to describe the RHIC data equally well



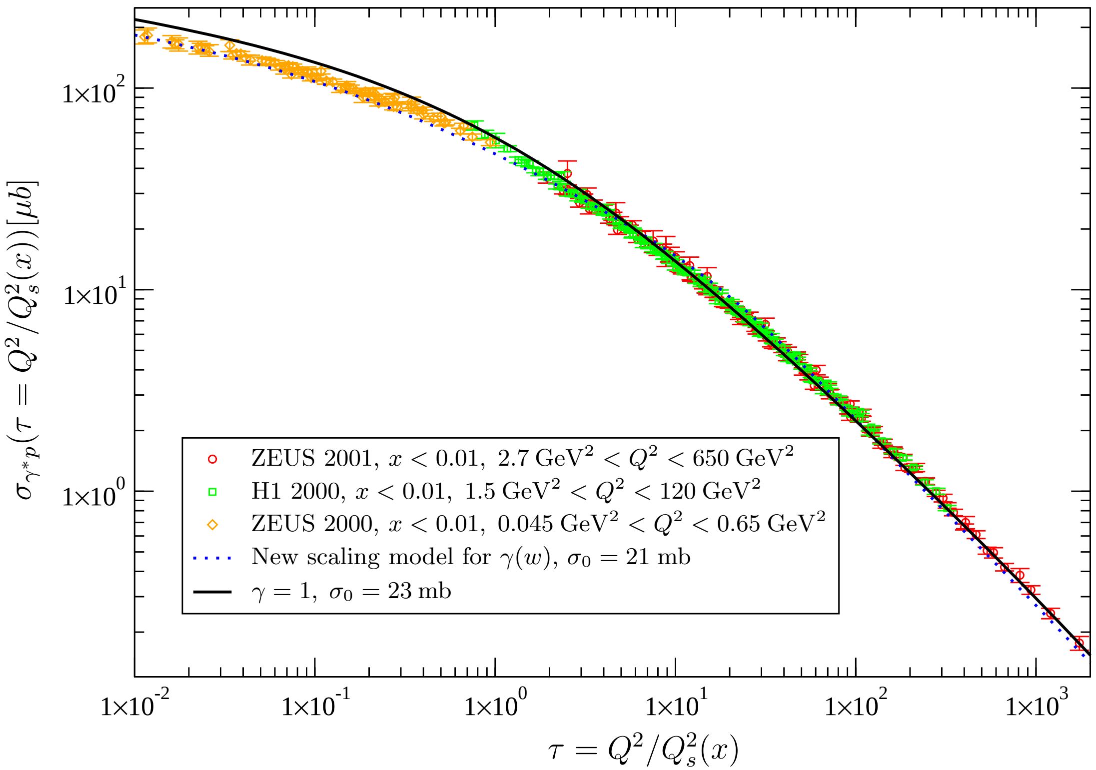
- **Forward region $y_h = 3, 4$**
  - Only region $q_t = \mathcal{O}[Q_s]$ where $\gamma(w) \approx \gamma_1$ probed
  - Even γ_1 hardly constrained
- **Central Region $y_h = 0, 1$**
  - Probe large $w = q_t/Q_s$ rise of γ $1-\gamma(w) \propto 1/w^a$
  - Logarit. rise $1-\gamma(w) \propto 1/\log w$ im-compatible with data
- Note, that a whole y_h range has to be probed to establish scaling violation $\gamma(w, y)$
  - At one y_h a range of $y = 2y_h + \log 1/x_1$ is probed.
  - However, for a single y_h one can always define a scaling $\gamma(w) \Leftrightarrow \gamma(w, y)$
- Region where DHJ/BFKL model works a constant $\gamma(w) \approx \gamma_1$ would already work
  - γ_1 doesn't have to be $\gamma_s \approx 0.628$

New model and DIS

- Check whether new model is compatible with DIS data using dipole cross section

$$\sigma_{\mathrm{DP}}(rQ_s(x)) = \sigma_0\, N_\gamma(rQ_s(x)) = \sigma_0 \left(1 - \exp\left[-\tfrac{1}{4}(r^2 Q_s^2(x))^{\gamma(Q/Q_s(x))}\right]\right)$$

- $Q^2 \gg Q_s(x)^2$: same predictions as in GBW model ($\gamma = 1$)
- Region $Q^2/Q_s^2(x) \approx 10 - 100$: requires smaller σ_0 (21 mb instead of 23 mb)
- Satura. region $Q^2/Q_s^2(x) \ll 1$: smaller γ suppresses $\sigma_{\gamma^* p}$ requires smaller quark masses



4. LHC predictions

- RHIC, region where DHJ/BFKL model fails: x_2 is not very small

- LHC larger energies: small-x_2 extends to larger p_t-range
  $\Rightarrow$ slower (BFKL) fall-off of p_t distribution manifests in small x_2 region

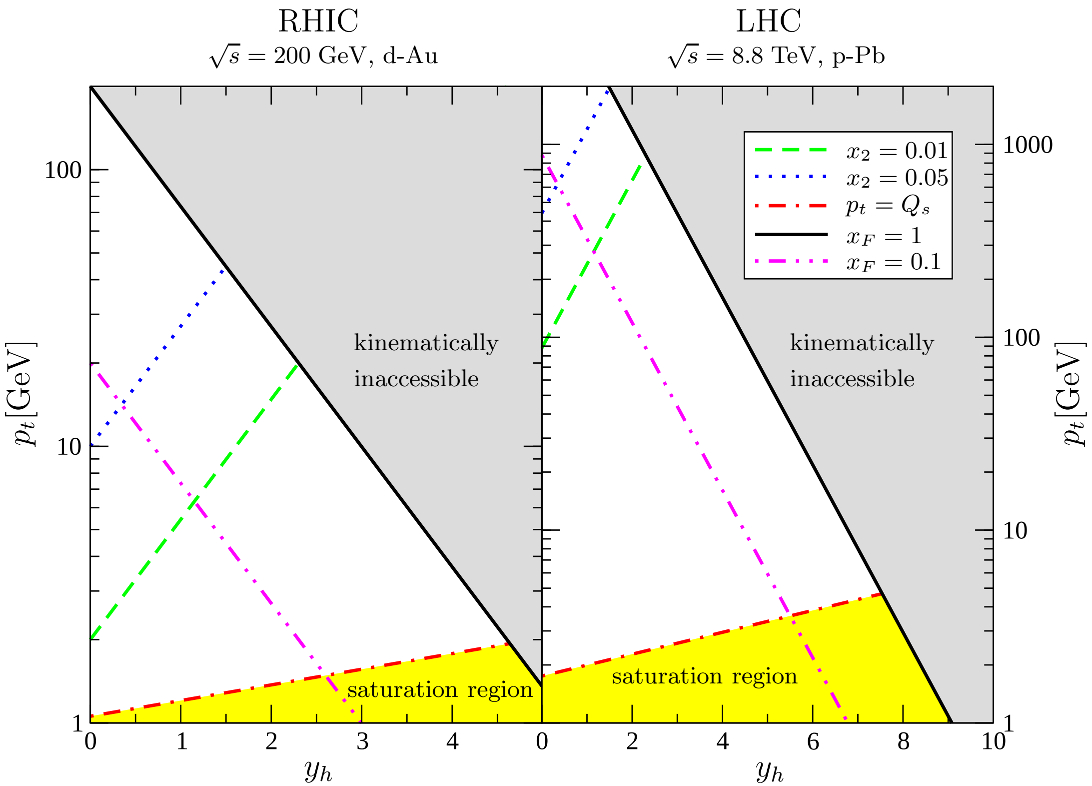
- Small x_2 in terms of p_t and y_h

– $x_2 \lesssim 0.01$: DHJ works at RHIC

- Saturation region $p_t \leq Q_s(x_2)$.

- d-Au: $A_{\text{eff}} = 18.5$, $\sqrt{s} = 200\,\text{GeV}$
  p-Pb: $A_{\text{eff}} = 20$, $\sqrt{s} = 8.8\,\text{TeV}$

- Dominant contribution to conv. integral, region x_1 close to x_F
  $\Rightarrow x_2 \approx p_t/\sqrt{s}\exp(-y_h)$.



Hadron production at LHC

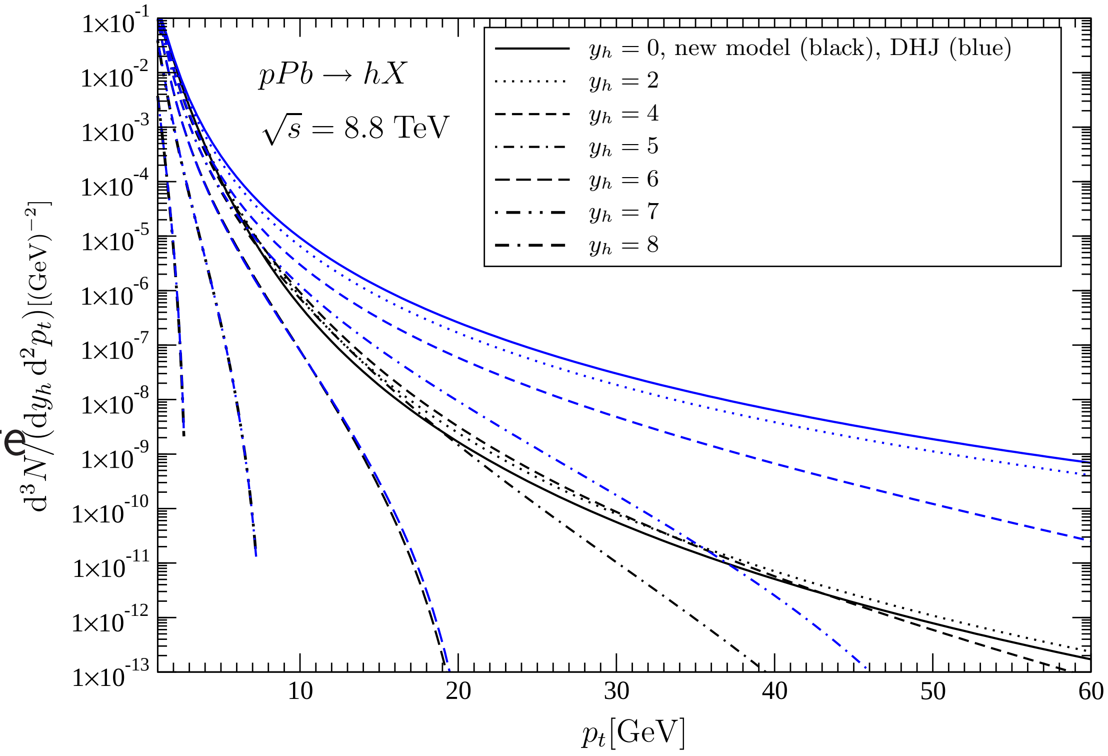
- Predictions for p-Pb scattering at $\sqrt{s} = 8.8\ \mathrm{TeV}$ in **small-x_2** region

- **Small p_t**, similar predictions of DHJ and new scaling model

- **Forward region $y_h \approx 7 - 8$**: Like at RHIC $\gamma \approx \gamma_1 \Rightarrow$ same predictions in the models

- **Large region of small x_2** where predictions are clearly different

– p_t slopes at moderate y_h's $\Rightarrow$ discrimination between DHJ and our model in small-x region



- Very similar predictions for p-p scattering at $\sqrt{s} = 14\ \mathrm{TeV}$

- **Predict. of our model and BFKL inspired model clearly differ. at small x**

  – LHC offers a clear test of BFKL features ($\gamma_1 \approx \gamma_s$, logarithmic rise of γ)

Jet Production

- **Unlike in DIS**, scaling dipole amplitude does not imply scaling cross section
- Problem less involved for **jet production**
  - **Jet cross section** does not involve any fragmentation functions $D_{h/(q,g)}(x_F/x_1, p_t^2) \to \delta(x_F/x_1 - 1)$

$$\frac{dN_h}{dy_h d^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \left[\sum_q f_{q/p}(x_F, p_t^2)\, N_F\,(p_t, x_2) + \; f_{g/p}(x_F, p_t^2)\, N_A\,(p_t, x_2) \right],$$

  - where $x_F = p_t/\sqrt{s}\,\exp(y_h)$ and $x_2 = x_F \exp(-2y_h) = p_t/\sqrt{s}\,\exp(-y_h)$.
  - Still complications from non-scaling parton distribution
    $\Rightarrow$ even for scaling $N_{A,F}$, no scaling in $\mathrm{d}N/(\mathrm{d}y_h\, \mathrm{d}p_t^2)$
  - Gluon (quark) dominance
    $\Rightarrow$ $(p_t^2\, dN_h/dy_h d^2 p_t)/f_{g(q)/p}(x_F, p_t^2)$ would be a function of $p_t/Q_s(x_2)$ only
- However, range of gluon dominance presumably even at LHC to small to establish geometric scaling (violation) directly in this way

Conclusion

- **Scaling model of dipole scattering amplitude $N(r, x)$ describes RHIC data**
  - $\Rightarrow$ RHIC d-Au data completely compatible with geometric scaling
  - Models (DHJ) inspired by small-x evolution fail at mid-rapidity
  - There, a faster rise of γ is required
  - Both models work for forward rapidities
  - There, also a constant $\gamma(w) \approx \gamma_1$ works

- **Model also compatible with small-x DIS data**

- **Differences between our model and expectations from small-x**
  - No scaling violation
  - Phenomenologically more important, faster fall-off of p_t distribution

- **New insight to be expected from LHC**
  - Different fall-off of the p_t distribution shows up where x is still small
  - Allows to test BFKL-like rise $\propto \log q_t/Q_s$ at small x